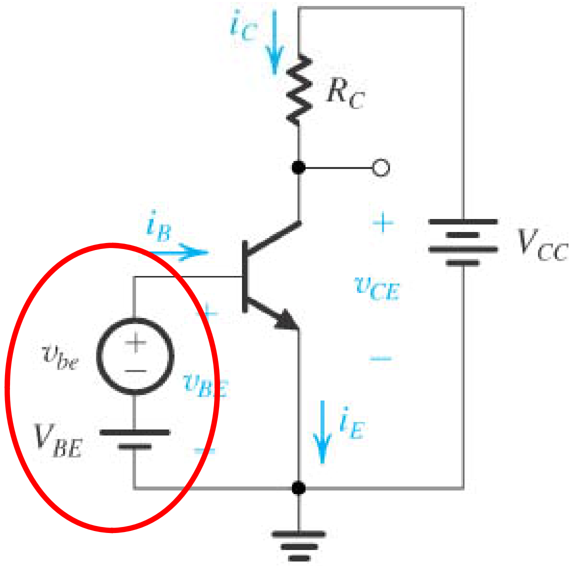
Lect. 12: BJT Small Signal Model (4.4.4 & 4.6.3 in Razavi)



How much change in i_C for a small change in v_{BE}?

$$\upsilon_{BE} = V_{BE} + \upsilon_{be};\; V_{BE} >> \upsilon_{be}$$

$$i_C = \alpha i_E = \alpha I_S (e^{\upsilon_{BE}/V_T} - 1) \sim I_S e^{\upsilon_{BE}/V_T}$$

$$= I_S e^{(V_{BE}+\upsilon_{be})/V_T}$$

$$= I_S e^{(V_{BE}/V_T)} e^{(\upsilon_{be}/V_T)}$$

$$\sim I_S e^{(V_{BE}/V_T)} (1 + \frac{\upsilon_{be}}{V_T})$$

$$= I_C + i_c \qquad \text{where } i_c = \frac{I_C}{V_T} \cdot \upsilon_{be}$$

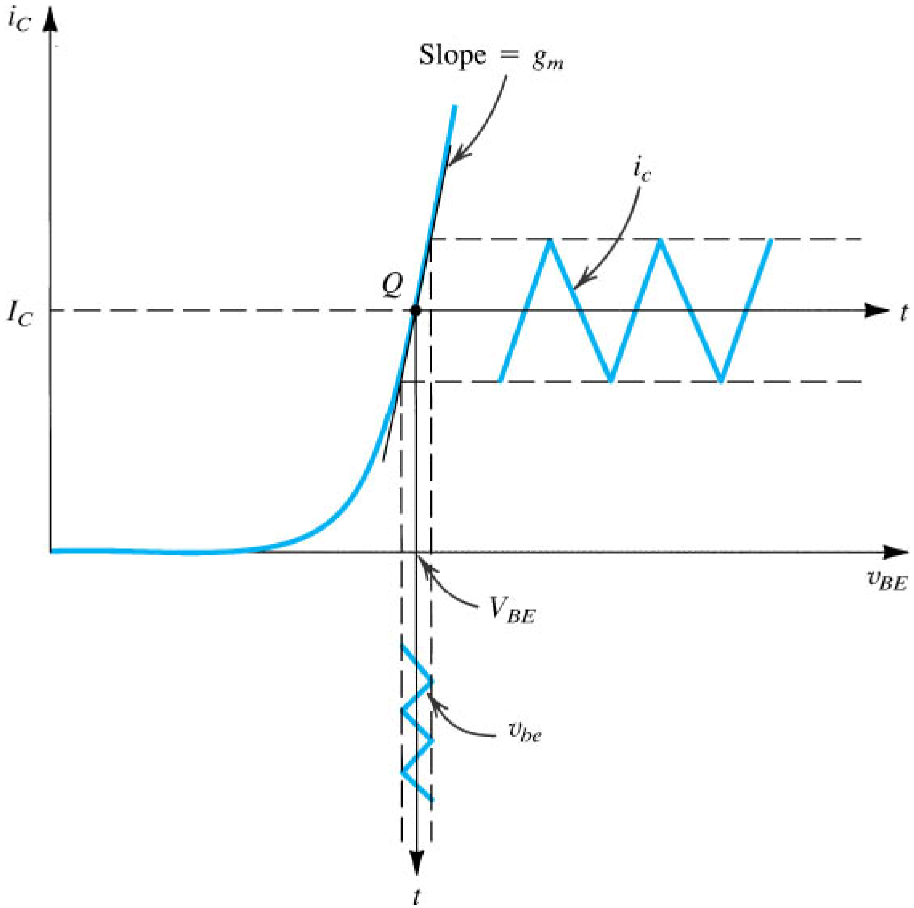
$$i_c = g_m \cdot \upsilon_{be}$$

$$(g_m = \frac{I_C}{V_T} : \text{ transconductance})$$

V_{BE}, I_C : Large signal, bias

υ_{be}, i_c : small signal, small changes

Lect. 12: BJT Small Signal Model



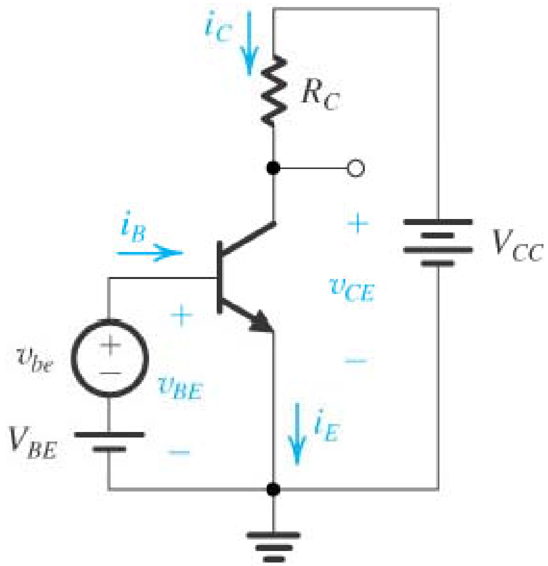
$$i_C = I_S e^{\upsilon_{BE}/V_T}$$

$$= I_S e^{(V_{BE}+\upsilon_{be})/V_T}$$

$$= I_C + i_c$$

$$i_c = g_m \cdot \upsilon_{be} \quad (g_m = \frac{I_C}{V_T})$$

Lect. 12: BJT Small Signal Model



(a)

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{g_m v_{be}}{\beta}$$

$$i_b = \frac{g_m}{\beta} v_{be} = \frac{i_c}{\beta}$$

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{g_m v_{be}}{\alpha}$$

$$i_e = \frac{g_m}{\alpha} v_{be} = \frac{i_c}{\alpha}$$

$i_B = I_B + i_b = ?$

$i_E = I_E + i_e = ?$

- All small signal parameters have **linear** relationship

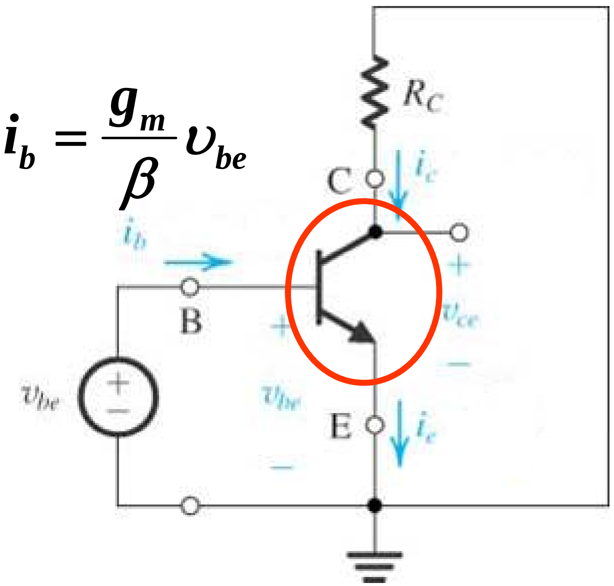
- Their values depend on large signals

Lect. 12: BJT Small Signal Model

Small-signal (linear) circuit Model

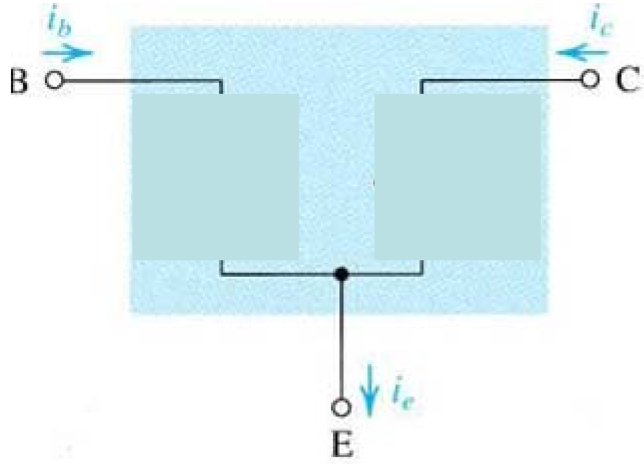
$$i_c = g_m \cdot \upsilon_{be}$$

$$i_b = \frac{g_m}{\beta}\upsilon_{be}$$



$$i_e = \frac{g_m}{\alpha}\upsilon_{be}$$

Hybrid-π model

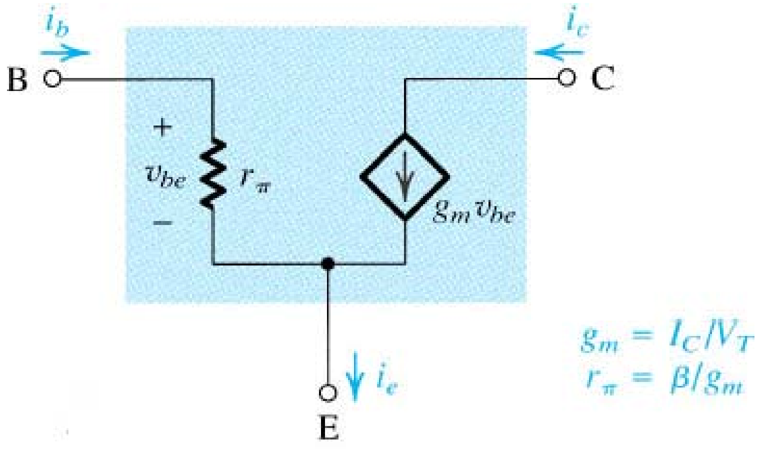


$$r_\pi \equiv \frac{\upsilon_{be}}{i_b} = \frac{\beta}{g_m}$$

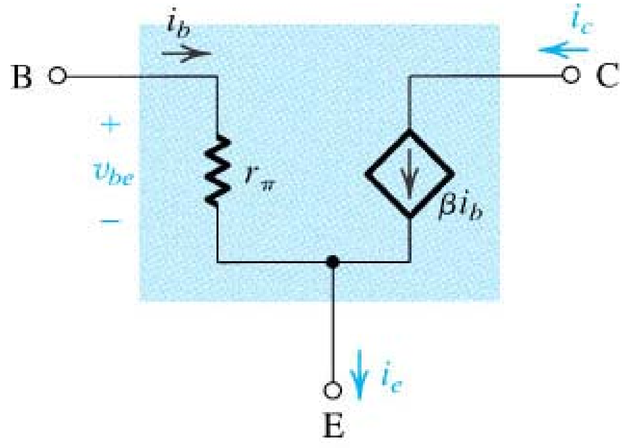
$$= \frac{I_C}{I_B} \cdot \frac{V_T}{I_C} = \frac{V_T}{I_B}$$

Lect. 12: BJT Small Signal Model

Hybrid-π model



(a)

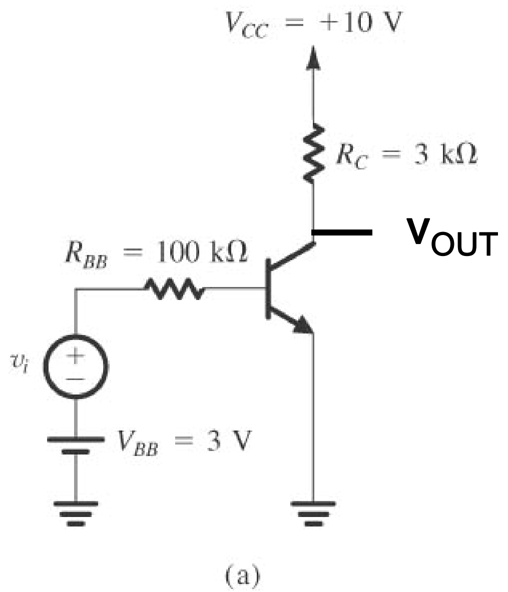


(b)

$$g_m v_{be} = g_m(i_b r_\pi)$$
$$= (g_m r_\pi) i_b$$
$$= \beta i_b$$

Lect. 12: BJT Small Signal Model

Small signal voltage gain, v_{out}/v_i ?



(β = 100)

DC Analysis: Assume v_i = 0

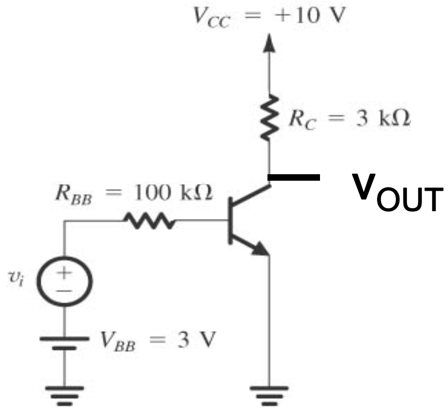
$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}} = \frac{3 - 0.7}{100} = 0.023 \text{ mA}$$

$$I_C = \beta I_B = 2.3 \text{ mA}$$

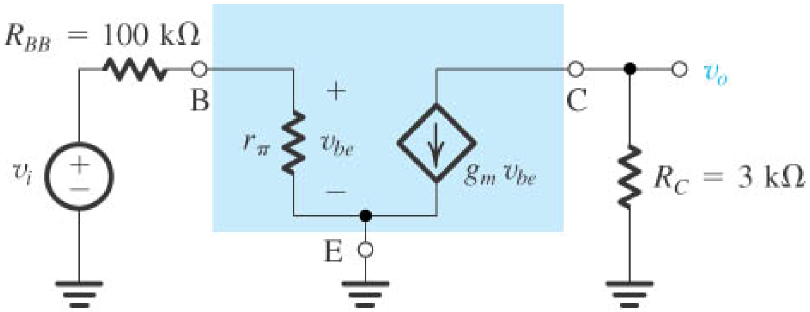
$$V_C = V_{CC} - I_C R_C$$

$$= 10 - 2.3 \times 3 = 3.1 \text{ V}$$

Small-signal analysis



(a)



(c)

$$g_m = \frac{I_C}{V_T} = \frac{2.3\text{ mA}}{25\text{ mV}} = 92\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{92} = 1.09k\Omega$$

$$\upsilon_{be} = \upsilon_i \frac{r_\pi}{r_\pi + R_{BB}}$$

$$= \upsilon_i \frac{1.09}{101.09} = 0.011\upsilon_i$$

$$\upsilon_o = -g_m \upsilon_{be} R_C$$

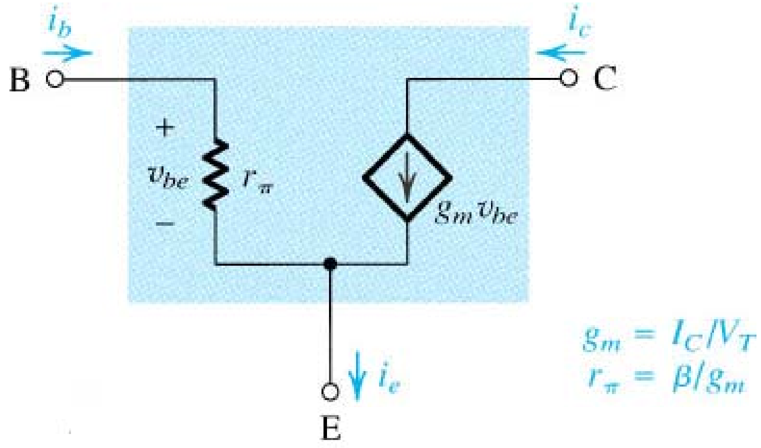
$$= -92 \times 0.011\upsilon_i \times 3 = -3.04\upsilon_i$$

$$A_\upsilon = \frac{\upsilon_o}{\upsilon_i} = -3.04$$

Lect. 12: BJT Small Signal Model

Is the small signal model good enough?

Hybrid-π model

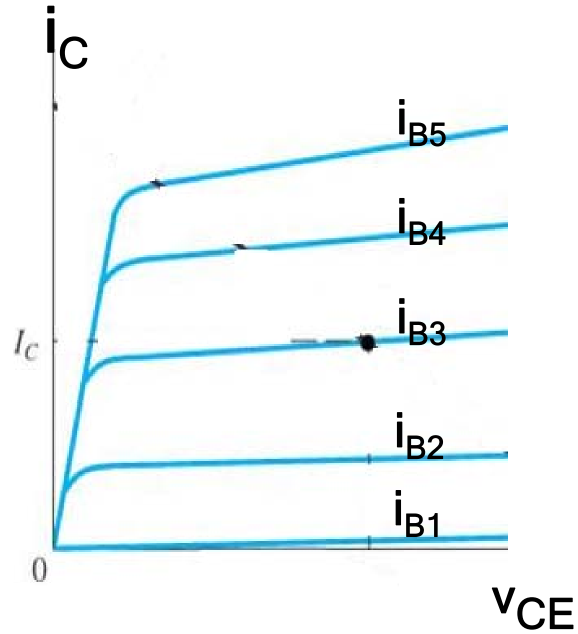


(a)

$$i_C = I_S e^{v_{BE}/V_T}$$

i_C should be independent of of v_{CE}

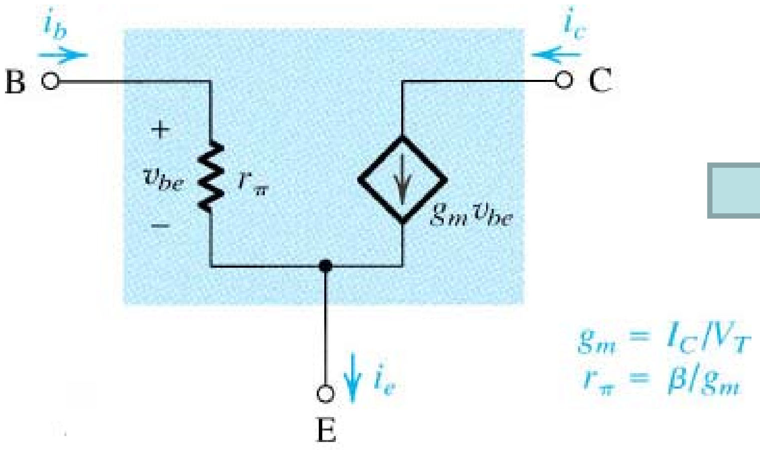
But in real BJT devices,



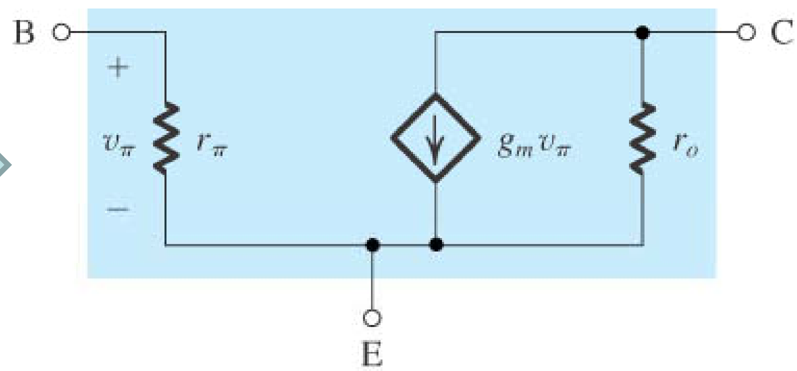
Increase in i_C with v_{CE} due to Early Effect!

How can we model this?

Hybrid-π model



(a)

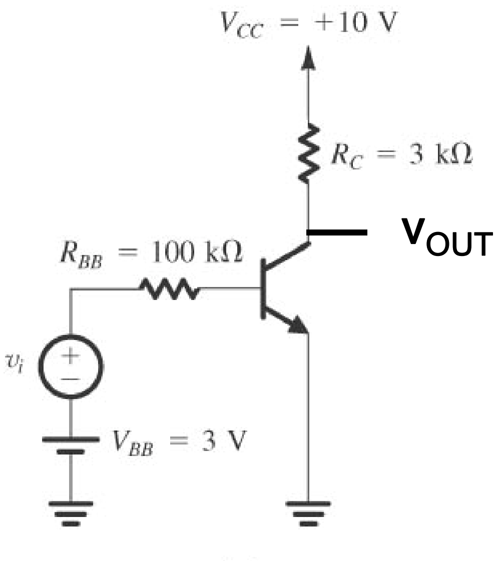


(a)

r_o depends on BJTs

Typically, in the order of 100kΩ

Lect. 12: BJT Small Signal Model



(a)

DC Analysis ($\beta = 100$, $r_0 = 100\text{k}\Omega$)

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}}$$

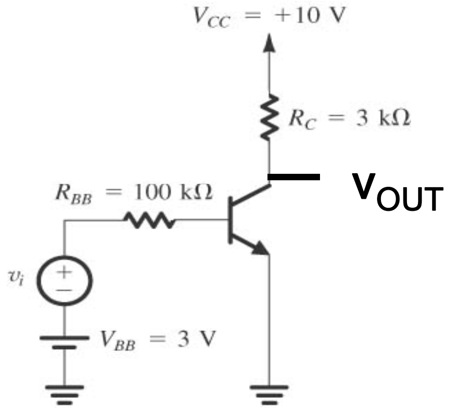
$$\sim \frac{3 - 0.7}{100} = 0.023 \text{ mA}$$

$$I_C = \beta I_B = 100 \times 0.023 = 2.3 \text{ mA}$$

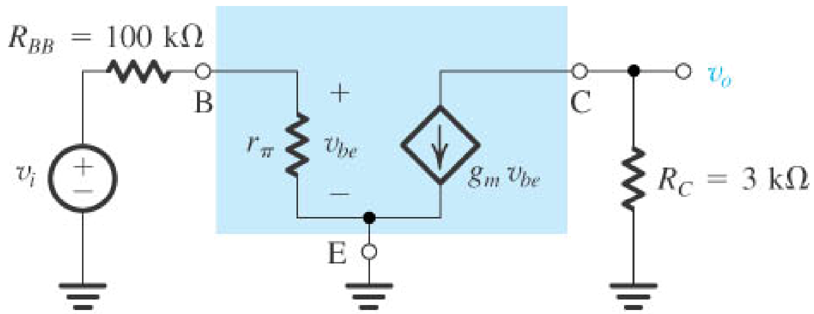
$$V_C = V_{CC} - i_C R_C$$

$$= +10 - 2.3 \times 3 = +3.1 \text{ V}$$

Small-signal analysis



(a)



(c)

$$g_m = \frac{I_C}{V_T} = \frac{2.3\ \text{mV}}{25\ \text{mA}} = 92\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{92} = 1.09\ \Omega$$

$$\upsilon_{be} = \upsilon_i \frac{r_\pi}{r_\pi + R_{BB}}$$

$$= \upsilon_i \frac{1.09}{101.09} = 0.011\upsilon_i$$

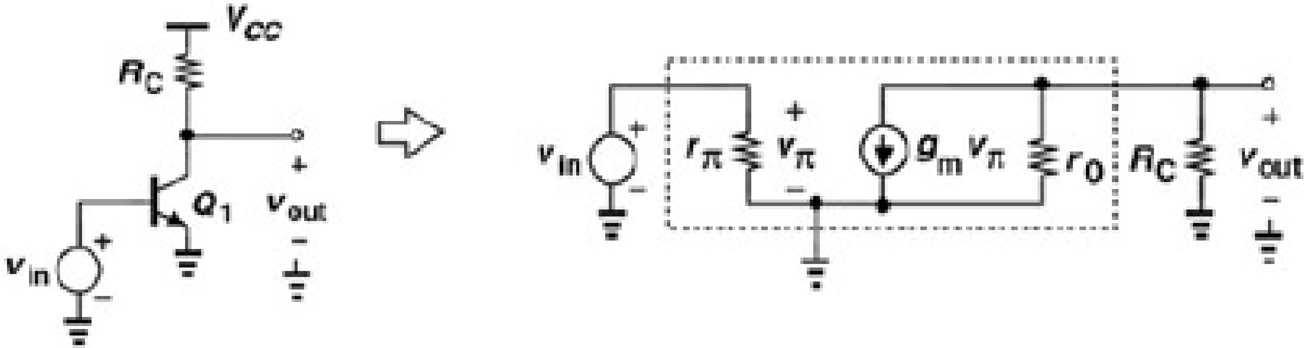
$$\upsilon_o = -g_m \upsilon_{be} (R_C // r_o)$$

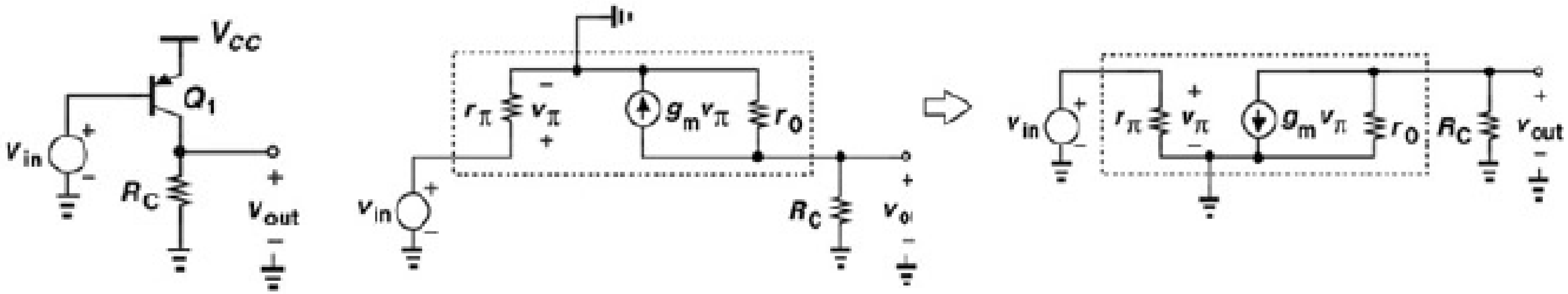
$$= -92 \times 0.011\upsilon_i \times (3//100) = -2.95\upsilon_i$$

$$A_\upsilon = \frac{\upsilon_o}{\upsilon_i} = -2.95\ \text{V/V}$$

Lect. 12: BJT Small Signal Model

Small signal model for PNP?





Small signal models for PNP and NPN are identical!